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DYNAMIC CALIBRATION OF PRESSURE PICK-UPS BY THE SHOCK-TUBE METHOD AND AUTOMATIC REDUCTION OF THE RESPONSE CURVES /27

Pierre Liénard, Jacques Hay and Maurice Planchais

ABSTRACT 14291 A

The dynamic calibration of pressure pick-ups with sensitivities that are too small to react to an ordinary noise field can be carried out with the pulse method. In the shock-tube,

a step of constant pressure between 10^4 and $12 \cdot 10^5$ Pa (1.4 and 170 psi) can be applied to the pick-ups during a few hundredths of a second. The ratio of the Fourier transform of the response given by the pick-up to that of the input signal is the transfer function of the pick-up.

The tube, the experimental method, and the equipment for recording the successive values of the response signal of the pick-up fixed at the bottom of the tube are described. This recording is introduced directly, by perforated tape, into the computers that give the frequency response of the pick-up.

Author

I. Introduction

Pressure pick-ups designed for measurements in the range of from one to several thousand pascals (Pa), with a frequency response of from a few cycles per second to several tens of kilocycles per second, have been described in a preceding article (Ref. 1). An instrument is only useful if it is calibrated. However, difficulties have been encountered in applying an alternating pressure having the above characteristics of amplitude and frequency to these pick-ups.

Whereas the static calibration (zero frequency) requires a standard technique which is applicable to pick-ups and which allows measurement of d.c. pressures, the dynamic calibration requires an analysis of the response of these pick-ups to a pressure pulse or a pressure step, because it is not possible to permanently and economically produce high intensity sinusoidal pressures of known values. On the other hand, it is possible to produce with a shock tube a unit step of pressure of calculable height and of sufficient duration so that the low frequencies of its frequency spectrum are well known (Refs. 2 and 3).

It is known that the principal characteristic of a shock tube, which is of interest to us, is that it produces a unit step pressure of calculable height with an extremely small rise time (of the order of 10^{-9} sec) (Ref. 2). This is practically instantaneous as compared with audio effects. Also, part of this characteristic is that the pressure is subsequently constant for a relatively short time (in regard to equipment); however, this time is shorter the longer the tube. The pressure jumps that can be set up vary widely and can be high, provided the tube is constructed to sustain high pressures.

The shock tube mentioned here has a total length of 25 m. It is designed to produce, on the investigated pick-up, a pressure jump with a value that can be chosen between 10^4 and $12 \cdot 10^5$ Pa, with the possibility of still increasing this limit. The initial pressure is presently the atmospheric pressure, but in the future it will be possible to have a partial vacuum for the initial pressure. Under these conditions, the plateau time is approximately from 10 to 80 msec.

The investigated pressure pick-up is secured to the closed end of the tube expansion chamber, with suitable precautions being taken to isolate the wall and cavity vibrations. The pick-up receives the pressure /28 shown in Figure 2(a). The transmitted signal would be that of Figure 2(a) if the pick-up were perfect; however, it appears more like that in 2(b), (c), or (d), displaying its own response time, its own frequencies, etc.

The pick-up transfer function is the complex ratio of the Fourier transforms of the output function s(t) to the input function p(t).

The unit step p(t) is, however, of limited duration (for example, 60 msec with a tube 25 m long). The calculation of the corresponding integrals is only possible if the output function reaches a constant value at the end of this duration (Figures 2(b) and (c)). The case of Figure 2(d) cannot be immediately analyzed because the complete response of the pick-up is not known. It is therefore necessary to obtain a pressure plateau as long as possible.

A study is under way to analyze the almost stabilized transient responses in the case where the average value of the transient response is close to a decreasing exponential, with the amplitude of the fluctuation about this exponential also decreasing (See Section VI).

II. Description of the Constructed Tube

This tube is made up of solderless segments of steel tubing having an inside diameter of 104 mm, ending in soldered flanges which permit them to be assembled end to end in any sequence; the whole set rests in a horizontally movable position (Figure 1).

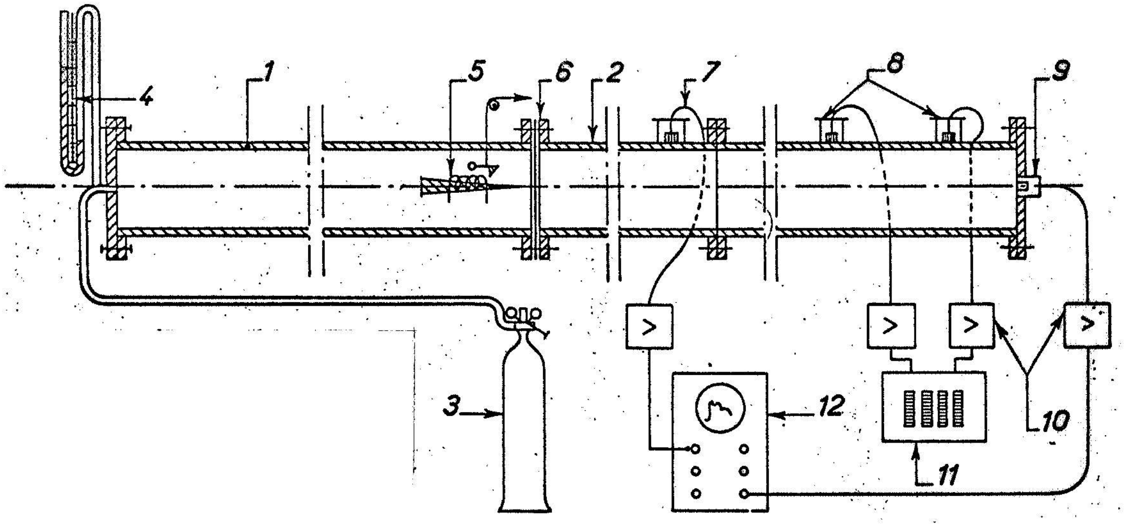

Figure 1. Schematic diagram of the whole apparatus

1, Compression chamber; 2, Expansion tube; 3, Compressed air; 4, Manometer; 5, Hammer; 6, Membrane; 7, Triggering pick-up; 8, Pick-ups to measure the velocity; 9, Pick-up under study; 10, Amplifiers; 11, Counter; 12, Oscilloscope (recorder).

The last segment of the compression chamber is closed by a plate with a fitting for compressed air. The segment which is to receive the membrane separating the compression chamber from the expansion tube contains a mechanical hammer. The latter is triggered from the outside simply by a wire. We had to eliminate electromagnetically controlled hammers because they induce parasitic currents in the sometimes very sensitive measuring devices.

The membrane (rhodium, aluminum or other material) is first inserted between two aluminum flanges having a hole the size of the tube. The flanges are then pressed between the tube segments. /29

In the expansion tube, one of the segments supports a piezoelectric microphone (barium titanate, type 20 H 42), flush with the wall, and mounted on a flexible plastic cylinder so as to isolate the mechanical vibrations of the wall. When the shock wave passes, the signal generated by this pick-up serves to trigger the scope.

The last segment of the expansion tube is 3 m long and supports 2 pick-ups, identical with the first one and separated by 2 m. The

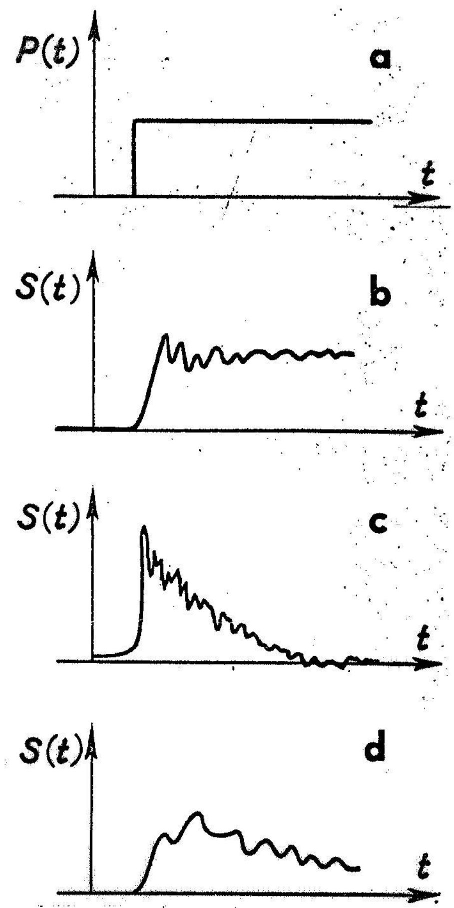

Figure 2. Input superpressure and response
a, Unit step of pressure; b, response of a pick-up which transmits the d.c. pressure; c, response of a pick-up which does not transmit the very low frequencies; d, unknown response at the end of the experiment.

corresponding signals produced when the shock wave passes are sent to another pick-up and permit the speed of the shock wave at the end of its trip to be measured; i.e., just before it reaches the investigated pressure pick-up.

The shock wave is reflected by the last plate of the expansion tube, which has at its center the investigated pick-up whose active face must be flush with the interior wall. The arrangement must be determined for each pick-up. It is possible to place two or more pick-ups on this plate for comparative purposes if there is sufficient room. One must avoid placing these pick-ups too close to the cylindrical wall because of possible perturbations. In practice the pick-ups should be within a circle of

5 cm diameter around the axis of the tube. If the pick-up is sensitive to vibrations, it must be mechanically isolated without introducing any discontinuity in the surface of the end plate. The whole arrangement includes compressed air tanks, manometers for measuring the initial pressure (before striking the membrane), amplifiers for the piezoelectric pick-ups to trigger and measure the speed, electronic gates and counter, and a singly triggered display oscilloscope to be controlled by the first pick-up mentioned previously (Figure 1).

After having investigated and completed the attachment of the pick-up to the terminal plate, the test must be prepared by selecting the lengths of the compression and expansion tubes and the pressure in the compression tube to obtain the desired pressure step and plateau time on the pick-up. The nature of the membrane suitable for the selected pressure must also be chosen, and the position of the triggering pick-up must be selected to obtain on the display a horizontal trace representing the initial pressure, and a suitable scaling of the response signal.

The standard formulas for shock tubes enable the adjusting parameters to be obtained with tables and computational charts.

III. Effect Employed: Nomenclature

The object of the shock tube is to produce, by the rupture of the membrane, a plane shock wave perpendicular to the tube axis, to be followed by a constant velocity and constant pressure flow which provokes at the closed bottom of the tube an abrupt jump of pressure to a pressure constant within a small time interval.

This is true, provided the rupture of the membrane is sudden and instantaneous, and provided the end of the tube is rigid and has no leaks and no cavities. Finally, the length of the tube must be sufficient so that the shock wave becomes stable.

We recall here, schematically, how the tube works with a τ, X diagram (Figures 3 and 4). The nomenclature is: /30

space origin: the plane of the membrane
time origin: the rupture of the membrane
L: length of the compression chamber
l: length of the expansion tube
x: abscissa of the point along the tube

$X = \frac{x}{L}$

t': time

Type
Proof
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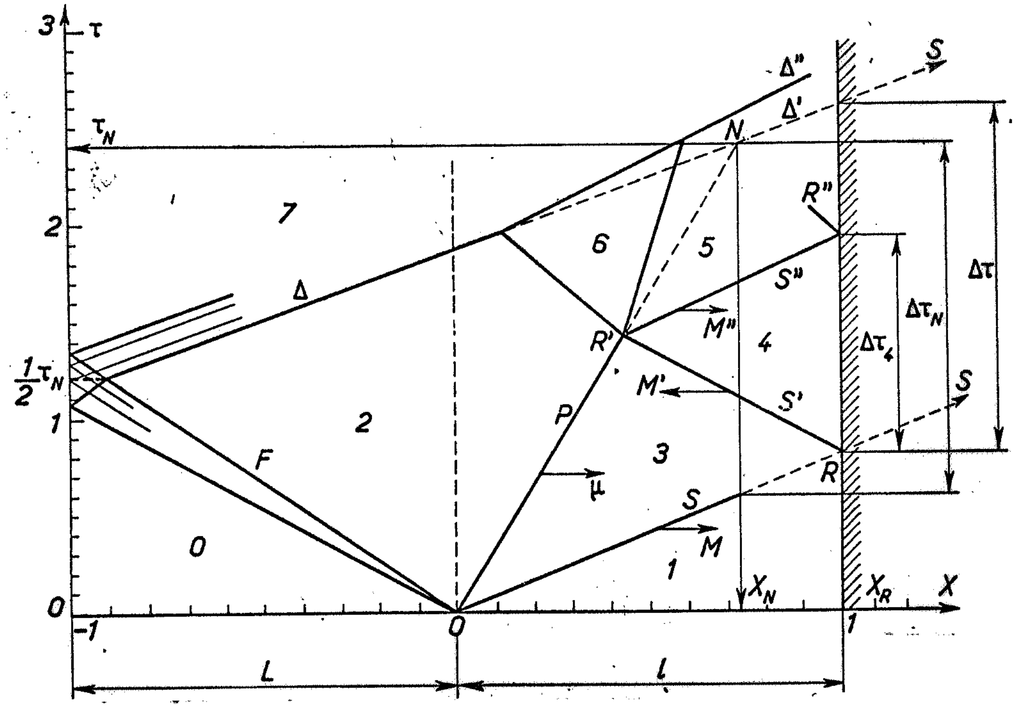

Figure 3. Wave propagation diagram

△, Wave front of the reflected expansion wave;
F, expansion pencil;
S, shock wave;
S', reflected shock wave;
S", reflected shock wave;
P, plane of discontinuity.

τ: reduced time $= \frac{a}{L} \cdot t$

a: sound velocity in air at the initial temperature

γ: ratio of the specific heats

p_i: pressure of the gas in the i region of the space-time diagram of Figure 3.

In the X, τ diagram, the various waves determine several regions (Figure 3) which determine the running index i above. The most important parameter, with respect to which the other quantities are simply expressed, is the absolute velocity M, in mach numbers, of the initial compression wave (between the regions 3, 1 of Figure 3).

Similarly, we designate the initial pressure ratio by P and the pressure jump at the end of the tube by Δp:

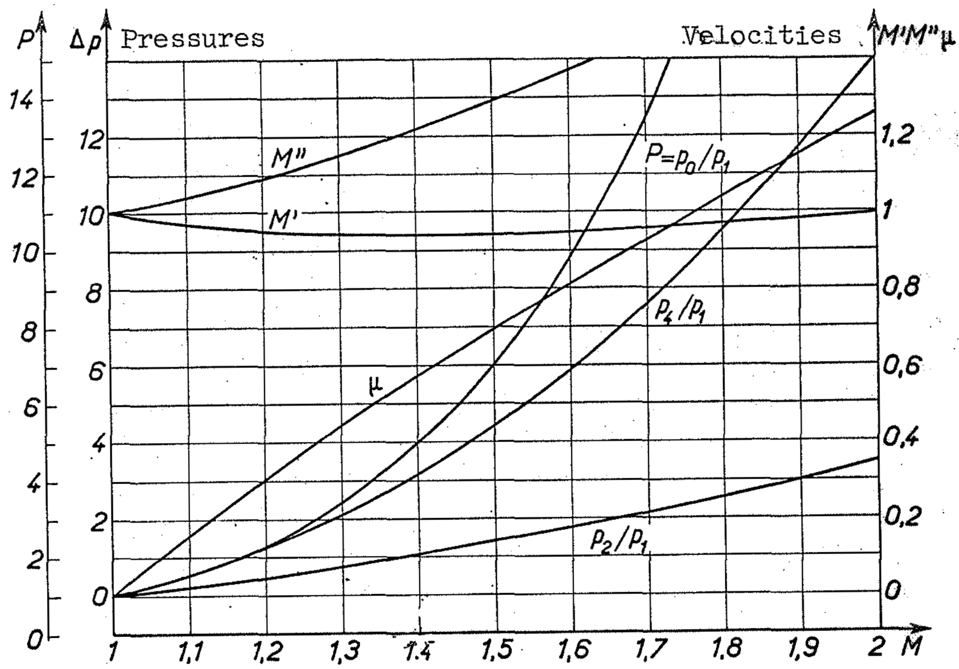

Figure 4. Graphs for the calculation of the various quantities

$$P = p_0/p_1$$

$$\Delta p = (p_4 - p_1)/p_1.$$

The calculations are performed here for an air-to-air tube having a constant cross section, so that a is the same in the regions 0 and 1, and $\gamma = 1.4$.

The discontinuity, 2-3, of the diagram of Figure 3 is a separation between the masses of air situated initially on either side of the diaphragm. The velocities and pressures are the same for either side of this wave, but the temperature, and consequently the sound velocity, are not. This wave of discontinuity (as it is sometimes referred to) is not, properly speaking, a wave[1], but a temperature discontinuity which propagates with the gas molecules at the same velocity.

The pressure remains constant at any point in the tube, between the passing of the initial compression wave 3, 1 and that of the reflected expansion wave 7, 2 (provided this point is sufficiently far from the

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[1]"The surface of contact" as referred to in (Ref. 3).

bottom of the tube so that it does not see the reflected wave 4, 3 preceding it).

In spite of having the advantage of a fairly long time when the pressure is constant, this same position entails the disadvantage of having a too high pressure rise time because the latter is equal to the time the shock wave sweeps the surface of the pick-up (for example 14 μsec for a $M = 1.6$ wave on a pick-up of 8 mm diameter). This is acceptable only for a slow pick-up or for a pick-up of small size. The pressure at the wall is less than at the bottom of the tube (approximately half of it for a weak shock), and this can be an advantage in the case of a very sensitive pick-up (microphone), because it is not easy to produce a very weak shock wave since the rupture of the membrane is not rapid enough and the pressure p_0 is too small. Other means of rupturing the membrane should be tried so that they do not produce pollution and noise.

For various reasons, especially for ease of construction, the shock tube mentioned here was selected to be cylindrical, with the investigated pick-up located at the bottom of the tube. This tube was designed for the calibration of pick-ups of medium sensitivity (microphones for intense noises).

For every calibration experiment, we must first determine the pressure jump Δp which will be applied to the pick-up, to within a certain accuracy.

Certain formulas (which will be reviewed later) and certain graphs /31
(such as the one shown in Figure 4) permit us to calculate Δp and the time Δt during which the pressure is constant (plateau), so that the scope can be adjusted.

The measurement of the velocity M permits us to correct the value of Δp because the calculation of Δp from P does not take into account the damping which takes place during the displacement of the wave.

In Figure 3 the tube has a length of 2L. This figure shows the various wave reflections and refractions. This diagram therefore allows us to determine the time Δt_4 (calculated from $\Delta t_4 = \frac{L}{a} \Delta\tau_4$) during which the pressure is constant against the bottom of the tube (see Appendix I for the calculation of Δt_4.

If the tube is sufficiently long (Figure 3), the reflected expansion wave and the discontinuity wave meet at point N on the diagram, with the ordinates X_N, τ_N indicated in the numerical tables. The reflected expansion wave and the shock waves meet at point S (X_S, τ_S) which is generally very distant.

IV. Shock Tube Equations

The basic parameter involved here is the velocity M (relative to the sound velocity) of the initial shock wave in the expansion tube.

The rise pressure P as measured by an accurate manometer, the superpressures p_3 and p_4 and therefore the superpressure Δp, the velocity M' of the wave reflected at the bottom of the tube, the velocity μ of the discontinuity wave, and the coordinates of the points N and S of the diagram (Figure 3) are all obtained by standard calculations. Some of the results of these are given on the following table as examples.

The formulas are readily transformed into numerical tables and into graphs (Figure 4) to determine Δp and to construct the diagram for the chosen case.

This diagram enables the maximum value of the length X_R of the expansion tube to be found and the maximum duration of the plateau to be obtained.

Table 1. Some equations for the shock tube as a function of V

$$P = \frac{p_0}{p_1} = \frac{7\,M^2 - 1}{6}\left(\frac{6\,M}{1 + 6\,M - M^2}\right)^7$$

$$\frac{p_3}{p_1} = \frac{7\,M^2 - 1}{6}$$

$$\frac{p_4}{p_1} = \frac{7\,M^2 - 1}{3}\,\frac{4\,M^2 - 1}{M^2 + 5}$$

$$M' = -\frac{1}{3}\,\frac{M^2 + 2}{M}.$$

Note that the ratio of the superpressure at the bottom of the tube to the travelling superpressure can be put in the form:

$$\frac{p_4 - p_1}{p_3 - p_1} = 2\left[1 + (M - 1) + \frac{1}{6}(M - 1)^2 + \cdots\right]$$

in other words, for the weak shocks a reflection on a wall doubles the pressure, which is a standard result.

In the first approximation, we neglect the change in pressure due to the wave S" which is twice reflected (Figure 3). The preceding maximum depends then on the value $\Delta\tau$ along the plane X_R between the S and Δ waves. The maximum depends in this way upon L, and since the length of the tube is limited to $l + L = 25$ m, the possible maximum of Δt does not correspond to the maximum of $\Delta\tau$.

For medium shocks, the velocities of the S and Δ waves are relatively close together, and $\Delta\tau \approx \Delta\tau_N$. The time Δt is then found from $\Delta t = \frac{L}{a} \Delta\tau$. The exact determination of the plateau time $\Delta\tau_4$, taking into account the twice reflected wave, requires the calculation of its velocity M". This calculation can only be done by approximations (see Appendix I).

V. Calculation of the Transfer Function

The principle involved in this calculation was indicated at the beginning of this paper. We begin with an input function of the applied pressure (step of height Δp), and display the pick-up response on the oscilloscope. The forms obtained are half rectangular (or triangular) waveforms. In other words, a function such as p(t)(Figure 5(a)) leads to a response s(t) such as in Figure 5(b) or (c).

The areas under the two curves are well defined and can be calculated by the trapezoidal rule. Similarly, the products $s(t) \cos \omega t$ can be calculated for any given value of ω.

In practice, the ordinates s(t) of the experimental response curve must be read for values of t as close together as possible to find the frequency spectrum of this waveform at the high frequencies. /32

The oscilloscope display of the response is magnified, retraced for a sufficiently fine line to be obtained and the curve thus obtained (Figure 6) is then wrapped around a drum. A light spot is interrupted at each turn of the drum successively by the arbitrary x axis and by the curve. A photoelectric cell transmits a signal at these two instants, which controls the starting and stopping of a counter. The process is almost entirely automatic with the device described in Section VII. The numbers in the counter are stored for punching a coded tape, and the counter is returned to zero to receive the following numbers which represent the ordinate $s(t + \Delta t)$, etc.

The punched tape is then used for punching cards, and a relatively standard program determines for each value of the frequency given before

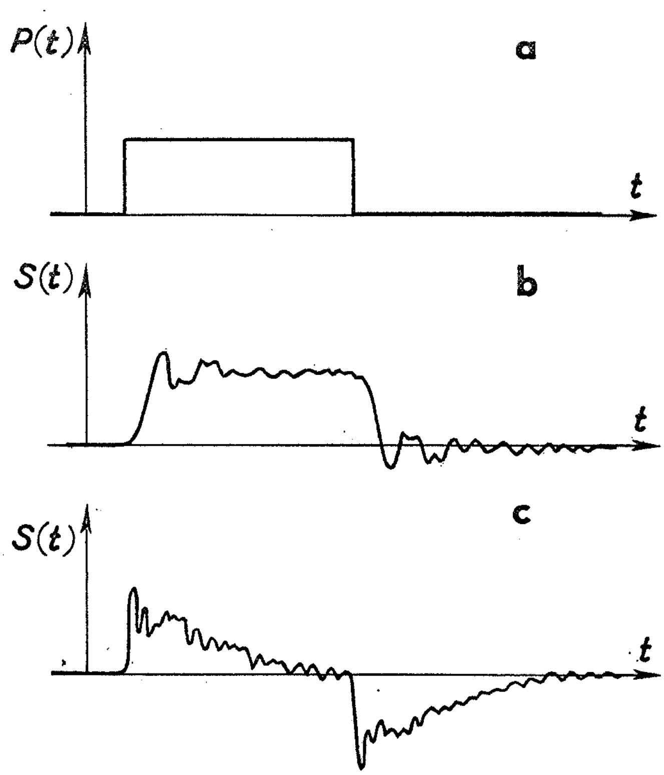

Figure 5. Input superpressures and responses. Representation as one period of periodic functions

the real and imaginary terms, and therefore the amplitude and the phase of the transfer function of the investigated pressure pick-up. Figure 7 shows an amplitude response curve obtained in this way.

Of course this calculation essentially presumes that a complete period of the responses like those in Figure 5(b) and (c) can be traced, and that the curve of Figure 2(d) is not usable as such. The advantage of having a pressure plateau as long as possible is obvious. Lengthening the plateau time Δt increases the chances of having a stabilized response at the end of the plateau, before all the reflected and refracted waves arrive to inextricably perturb the input wave function.

We have tried to use a response such as the one in Figures 2(d) or 8(b), which still contains sufficient information to be reduced as much as possible.

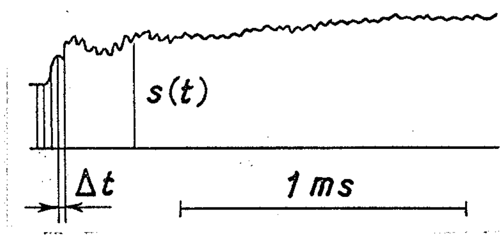

Figure 6. Example of a pick-up response

VI. Study of the Use of Certain Pressure Pick-Up Responses to the Unit Step Function, for the Case where the Pick-Up Does Not Return to Equilibrium at the End of the Pressure Plateau

Let θ be the time at the end of the pressure plateau—the pick-up output has still not returned to a state of equilibrium (Figure 8). For such a case, the problem is to extract from the output waveform s(t) all the information it contains, during the time interval when a known input function e(t) can be made to correspond to it; in other words, between the times 0 and θ. This information is useful for the response of the system to a discontinuous address, and therefore concerns the high frequency components of the spectrum.

We first considered placing an electric four-pole network at the output of the transducer, so that an output function s'(t), which returns to equilibrium at time $t = \theta$ (Figure 9), corresponds to the input function s(t). Since the series transducer, four-pole network can be analyzed by the standard method, we know that $Z'(\omega) = Z(\omega)Q(\omega)$ (in the interval where $Q(\omega) \neq 0$). Also, $Z(\omega)$ was deduced by finding the transfer function $Q(\omega)$ of this four-pole point by point. We abandoned this method because the determination of the four-pole necessitates knowing the function $Z(\omega)$ which we seek to define.

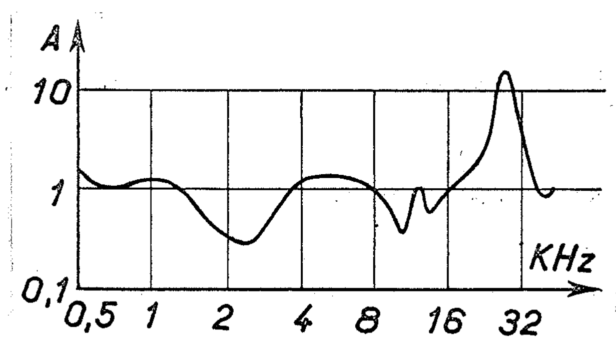

Figure 7. Amplitude vs. frequency response curve, calculated from the preceding response

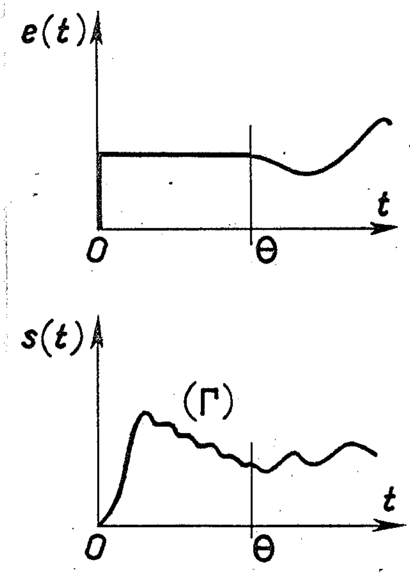

Figure 8. Input function differing from the step function after the time θ and incomplete response

The method we have adopted consists of mathematically extrapolating the function s(t) for times greater than θ. We designate the time function defined for $t > 0$ by $\sigma(t)$ to represent the output waveform corresponding to a unit step function of infinite duration which is applied at the transducer input. The calculation of the transfer function: /33

$$Z(\omega) = \frac{\frac{1}{2\pi}\int_0^\infty s(t)\, e^{-j\omega t}\, dt}{\frac{1}{2\pi}\int_0^\infty e(t)\, e^{-j\omega t}\, dt} = \frac{S(\omega)}{E(\omega)}$$

is split into two parts:

$$S(\omega) = Y(\omega) + Y'(\omega) = \frac{1}{2\pi}\int_0^\theta s(t)\, e^{-j\omega t}\, dt + \frac{1}{2\pi}\int_\theta^\infty \sigma(t)\, e^{-j\omega t}\, dt.$$

The first integral is calculated from the experimental function by a graphical integration method.

The second integral is found by calculation, depending on the mathematical form given $\sigma(t)$. In this way, a hand curve tracing is avoided.

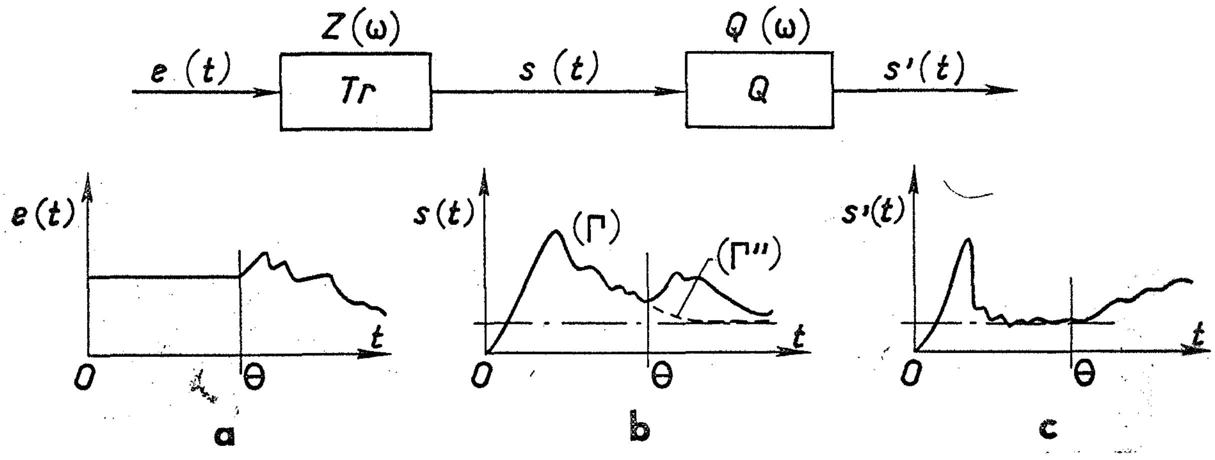

Figure 9. Transducer and auxiliary four-pole

On the other hand, the list of function types, which are useful in the extrapolation, is a limited one, and we cannot always take all the peculiarities of the experimental curve into account when its mathematical extrapolation is performed.

This latter disadvantage is largely compensated by the following advantage (Figure 10). The curve Γ is graphically extended by a curve Γ'. Since the curve reading machine can only make a maximum number N of readings, the experimental curve is known in this way to a number of points $n = N \frac{\theta}{\Theta}$ only, whereas in the case of a mathematical extrapolation, the experimental curve Γ is known to N readings. The rapid changes of the transducer are determined in this way with more accuracy and its behavior at high frequencies, which is sought here, is also known with more accuracy.

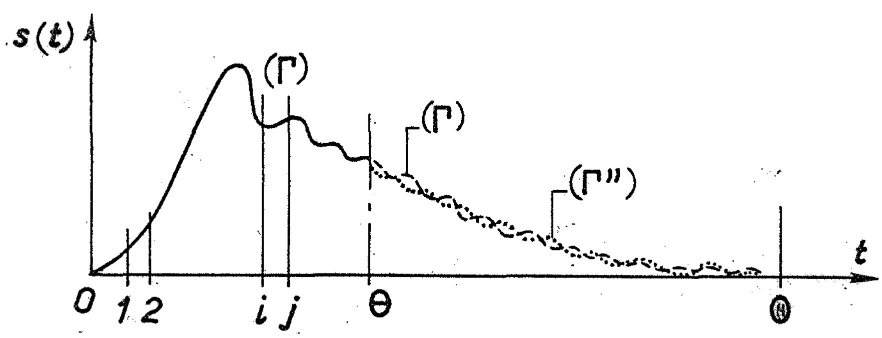

Figure 10. Extrapolation of the unfinished response at time θ

In the special case of pressure pick-ups, we have studied the most standard undamped response output waveforms. these can be of one of the four types shown in Figures 11 and 12.

It is seen that these curves are defined by the general equation:

$$y = a_1 e^{-\lambda_1 \tau} \cos \omega_1 \tau + a_2 e^{-\lambda_2 \tau} \cos \omega_2 \tau$$
$$\tau = t - \theta$$

where, depending on the case under consideration, certain parameters can be zero.

For each value of the frequency ω the transform:

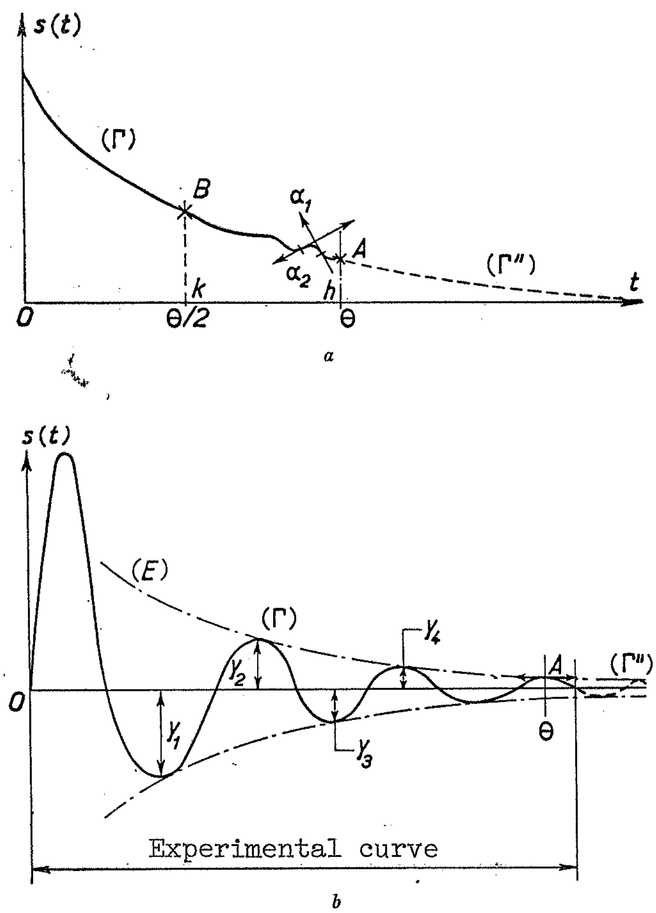

Figure 11. Examples of extrapolable functions

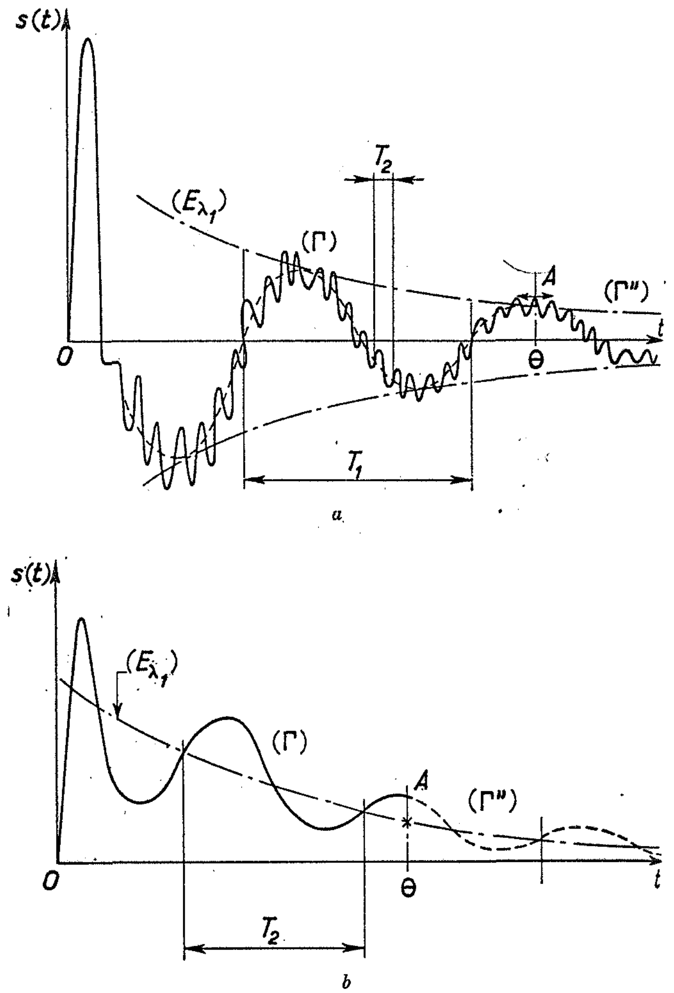

Figure 12. Examples of extrapolable functions.

$$Y'(\omega) = \frac{1}{2\pi}\int_{\theta}^{\infty} y(\tau)\, e^{-j\omega\tau}\, d\tau$$

is easily calculated. A program permits us to add the value taken by $Y'(\omega)$ to the calculation of:

$$Y(\omega)\ \frac{1}{2\pi}\int_{0}^{0} s(t)\, e^{-j\omega t}\, dt$$

which is done from the experimental curve. The transfer function $Z(\omega) = \frac{S(\omega)}{E(\omega)}$ is therefore determined in this way. /34

The determination of the limits of validity of this method, which sets the deviation between the mathematical extrapolation and the function which would be obtained in the case of an infinite step function, becomes rapidly complicated.

The limits of this method are studied by electrical analogs: Completely known electrical four-pole networks receive a unit step function which is truncated to lead to conditions analogous to those of the pressure pick-up.

VII. Device for the Automatic Reduction of the Response Curves

This device (Figure 15) automatically reads a curve, $y = f(x)$, traced in rectangular coordinates. It yields numerical values for the ordinates and abscissas of a certain number of points.

The numerical information obtained is transcribed on a punched tape, together with signal codes which are necessary for their use in computers.

1. Principle

The principle involved in the device is described in the following paragraphs (Figure 13).

The sheet with the curve to be analyzed is secured on a cylinder which rotates at a constant speed. The luminous flux emitted by a source lamp is directed and concentrated on an area of the cylinder, in front of which a photoelectric cell is located to receive the reflected light. The passage of the curve through the illuminated area is detected and transmitted by the photoelectric cell in pulse form.

For each value of the abscissa, the corresponding ordinate limits are characterized by two pulses. The first pulse triggers the time counter Y, while the second one stops it. The value obtained in this way is the numerical translation of the ordinate. The decades of the time counter are, an instant later, placed into correspondence with a memory device.

The abscissa is identified with a counter whose indication increases by one unit for each turn of the supporting cylinder. The sequence of the exploration cycle is defined by a rotating distributor which is driven at the same speed as the cylinder. It makes contact successively between each memory and the puncher, and controls the abscissa writer and the function signals. The duration of an exploration (one turn of the supporting cylinder) is about 900 msec. /35

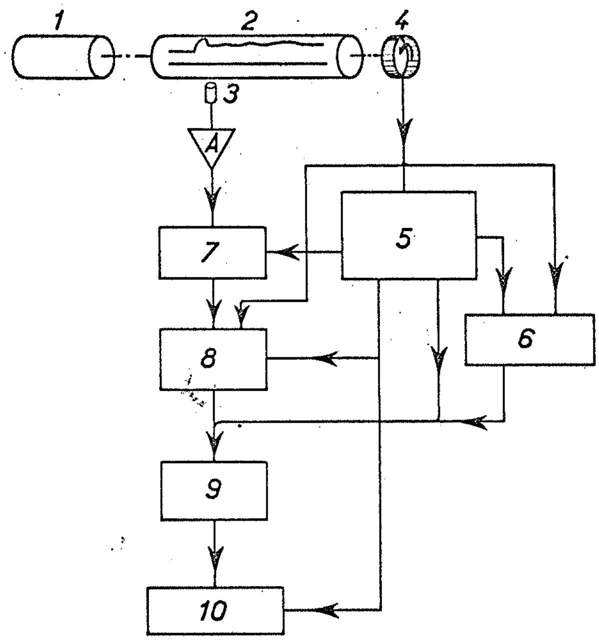

Figure 13. Block diagram

1, Motor;
2, Supporting drum;
3, Photoelectric cell;
4, Rotating distributor (20 positions);
5, Safety function control;
6, Counter X;
7, Counter Y;
8, Thyratron memory decades;
9, Coder;
10, Puncher.

The choice of the origin for the reduction of the curve is done by means of an adjustable decimal point. The command for transferring the information from the counter Y to the memory, and the return to zero of counter Y and its memories, are all distributed along one revolution and are repeated every revolution.

The continuous travel of the reading carriage is controlled by a worm gear. The distance between each point on the abscissa is 1/4 mm. The numerical value of a 1-mm ordinate corresponds to 43 units of the Y counter (1 unit ≈ 0.023 mm).

2. Curve Characteristics

The analysis of the curves (Figure 14) is only possible provided these curves have the following densities and dimensions:

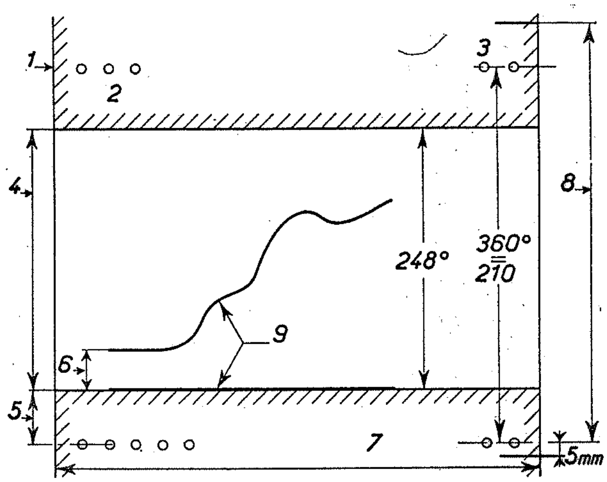

Figure 14. Curve display before readout

1, Guiding edge;
2, Counter cut-off zone;
3, Binding holes (overlap);
4, Useful maximum dimension: 145 mm;
5, Minimum distance from the lower edge to the axis: 35 mm;
6, Minimum distance from the curve to the axis: 5 mm;
7, Maximum width: 270 mm;
8, Maximum dimension: 235 mm;
9, Width of the trace: 1 mm ± 20 percent.

a. Solid color, mat, paper.

b. A straight line is provided to represent the x axis (the x axis and the curve can be traced in white on black paper or vice versa). The trace width must be 1 mm ± 20 percent.

c. Minimum ordinate is 5 mm.

d. Maximum ordinate is 145 mm.

e. Separation of at least 35 mm between the x axis and the lower paper boundary.

f. Maximum size of 24 x 27 cm.

g. Normal size of 21 x 27 cm. The use of smaller sizes is possible. The paper must be spotless and not be torn at the counting area.

Taking a density of zero for white paper, the density of black must /36
be equal to, or greater than, 0.4. The curves to be reduced can be either curves obtained from an oscilloscope with photographic paper, or

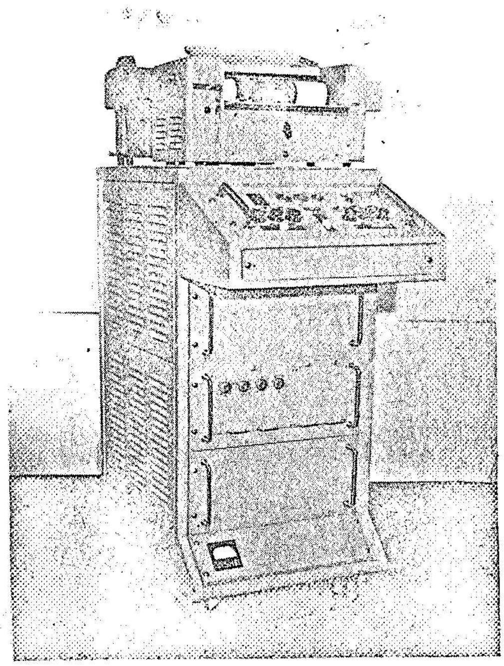

Figure 15. Automatic curve reader

the film or photograph of the oscilloscope trace, or any other type of reproduction. In the first two cases, it is possible to obtain by photographic treatment, the necessary difference in density between the background and the trace. For a negative film of the size 24 x 36, the curve which shows the evolution of the recorded effect is usable only after magnification. Reproduction can be either by photography, or by transparency tracing of the curve projected on a ground glass. A small scale magnification leads to too thick a trace, with unclear edges which must be retraced.

VIII. Validity Limit of the Method

1. Frequency Band

The principle involved in consideration of the signal shown in Figure 5 (which is the obtained square wave) is that the lowest frequency that can be analyzed is the fundamental frequency of the square wave; i.e., $\frac{1}{2\,\Delta t}$, where Δt is the plateau time obtained in the shock tube. A second advantage to having a longer plateau time is now apparent. With a plateau time of 50 msec, the analysis starts at the frequency of 10 cps. The extrapolation of the curves according to Section VI lowers the lowest frequency.

The highest frequencies that can be analyzed depend on the resolving power in reading the ordinates. If we assume that it takes five ordinate units to define a sinusoidal curve, or 4 reading steps, separated on the previously described equipment by 1/4 mm, then the reciprocal of the time scale in seconds per millimeter of the paper gives the highest frequency which can be analyzed without a large error. If the 50 msec of this paper are laid on 25 cm, this maximum frequency is 500 cps.

In this concrete example, we discover a bad application of the method, which can yield information at much higher frequencies; however, it is the reading which actually limits the frequency band at either end of the band.

An improvement would be to obtain during the test with the shock tube (or two identical tests) two recordings with different sweep speeds, one slow speed for analysis down to the lowest frequencies permitted by the plateau time, and a second high speed to use the information at the high frequencies. The merger of these two recordings by mechanographic computation after their separate computation is presently under study.

2. Amplitude

The upper pressure limit is determined by the tube strength and the quality of the joints. The present possibilities are largely sufficient for pick-ups employed in pressure fluctuations which come from audio stages, since these are not supposed to measure more than 1 or $2 \cdot 10^5$ Pa. A mechanical modification of the tube is under consideration to withstand pressures 10 to 20 times as great (for pick-ups like those in Ref. 4).

The lowest pressure is limited by the fact that below a certain value of P, the membrane tears poorly; i.e., $P > 1.1$ to 1.2 with a cellophane membrane.

In place of a shock wave in the tube, an acoustic wave can travel down the tube.

The minimum pressure jump (above the atmospheric pressure) is presently of the order of 10^4 Pa, or, in the logarithmic scale, 174 db. Since the most powerful sound installations can hardly furnish 140 db on a small area, the separation between these two values shows a region where the dynamic calibration problem is not solved (except by arrangements of the piston phone type, at very low frequencies only).

3. Precision

The error made is the sum of the errors of the recorder, the photocopy, the curve reader, and the value of the pressure plateau. It is difficult to estimate this sum. It seems to be of the order of $\pm$ 1 db ($\pm$ 12 percent). In the succession of operations, the reproduction or the photographic magnification seems to introduce the greatest errors.[1]

The definition of the responses as a function of frequency then depends on the number of calculated values as a function of frequency. Theoretically, there is no limit to the number of these calculations; however, in practice, the calculations made for too widely separated frequencies can overlook certain resonance peaks. We must repeat the mechanographic computation with closer frequency values around the probable value of these resonances.

IX. Conclusion

The calibration by the shock-tube method covers important needs in the field of wide-band pressure pick-ups. The preceding discussion also shows the still sharp limitations inherent in this method.

The curve reader, which considerably enhances the preceding method, can have many more applications, especially in translating graphical information to a digital computer.

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APPENDIX I /37

Successive Reflections of the Shock Wave at the Tube End

In the preceding calculations (Section IV), only the velocity M' of the wave reflected at the bottom of the tube has been taken into consideration, and the duration of the plateau $\Delta\tau$ (Figure 3) is only approximately known, provided the tube is long enough so that the point R' of intersection of this reflected wave with the plane of discontinuity is fairly close to point N.

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[1]The value of the pressure plateau Δp is given by calculation from the speed M, which is related to the pressure P of the compression chamber (equation p. 9). This relation is not completely checked by experiment for $P > 10$ ($M > 1.6$) (Ref. 3). In addition, because of the attenuation with the distance, it is preferable to measure M instead of calculating it from P, at least for the strong shocks and the great lengths X.

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In fact the constant pressure plateau ceases when shock wave S", reflected on this discontinuity wave a second time, returns to the bottom of the tube (Figure 3). Let M" be the velocity of the twice reflected wave, and $\Delta\tau_4$ be the (reduced) time separating the impacts of the initial wave S and the reflected wave S" against the bottom. The real plateau time is therefore:

$$\Delta t_4 = \frac{L}{a} \Delta\tau_4.$$

The calculation of M" was made assuming a ratio l/L such that the reflected shock wave S' meets first the discontinuity plane μ rather than the wave Δ'. A simple geometrical calculation leads to the following equation, as a function of the preceding velocities:

$$\Delta\tau_4 = X_R \frac{\mu - M}{\mu - M'} \frac{M' - M''}{M M''}.$$

The equations which permit the calculation of M" have been determined by the Aerodynamics Administration. This calculation was carried out by successive approximations. As a function of M we finally obtain:

$$M'' \simeq \frac{9 M^3 + 59 M^2 - 43 M + 47}{6 M (M + 11)}.$$

Expressing everything as a function of the single parameter M, the pressure plateau Δt_4 is given by:

$$\Delta t_4 \simeq \frac{l}{a} \frac{M' + 5}{M (7 M^2 - 1)} \left[1 + \frac{2 (M^2 + 2)(M + 11)}{9 M^3 + 59 M^2 - 43 M + 47}\right].$$

The calculation of this expression (the factor l/a being excluded) was listed and plotted (not shown here) for M going from 1 to 2.

It then becomes possible to determine, for each value of the superpressure P (Table 1 and curves of Figure 4), the optimum length l (assuming l + L = 25 m) in order to obtain the Δt maximum, while keeping a wave diagram geometry corresponding to the case in Figure 3.

APPENDIX II

Table II. Review of the units of pressure

System	Name	Symbol	Value
MKSA	Pascal	Pa	Newton/m^2
CgS	Barye		dyne/cm^2 = 0.1 Pa
	Millibar	mb	100 Pa
	Cm of water		g force/cm^2 = 98.1 Pa
M.T.S. (Meter, ton, second)	Pieze	Pz	1,000 Pa
	Bar	b	Hpz = 100,000 Pa
English	Pounds/ square foot	(lb/sqft)	0.488 g/cm^2 = 47.89 Pa
	Pounds/ square inch	(lb/sqin) (psi)	70.3 g/cm^2 = 6,896.4 Pa
Logarithmic scale	Decibel	db	$20 \log \frac{p}{p_0}$, $p_0 = 20{,}10^{-6}$ Pa (p is then the rms value of the pressure fluctuation)

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